

The Proof for FGJ_{self}

Chieri Saito and Atsushi Igarashi

Graduate School of Informatics, Kyoto University, Japan
{saito,igarashi}@kuis.kyoto-u.ac.jp

1 Proof of Theorem 4.1

In this appendix, we prove the subject reduction theorem of FGJ_{self} . The structure of the proof is similar to that of FGJ [1], beginning with weakening lemmas, followed by various substitution lemmas, showing the properties of the lookup functions. Since self type variables are a small extension, it is sufficient to update a few lemmas. In what follows, the metavariables Z and V range over type variables and types, respectively.

Lemma 1 (Weakening). *Suppose $\Delta, \bar{X} < \bar{N} \vdash \bar{N}$ ok and $\Delta \vdash U$ ok.*

1. *If $\Delta \vdash S < T$, then $\Delta, \bar{X} < \bar{N} \vdash S < T$.*
2. *If $\Delta \vdash S$ ok, then $\Delta, \bar{X} < \bar{N} \vdash S$ ok.*
3. *If $\Delta; X \vdash S$ ok-bound, then $\Delta, \bar{X} < \bar{N}; X \vdash S$ ok-bound.*
4. *If $\Delta; \Gamma \vdash e : T$, then $\Delta; \Gamma, x : U \vdash e : T$ and $\Delta, \bar{X} < \bar{N}; \Gamma \vdash e : T$.*

Proof. Each of them is proved by straightforward induction on the derivation of $\Delta \vdash S < T$, $\Delta \vdash S$ ok, $\Delta; X \vdash S$ ok-bound and $\Delta; \Gamma \vdash e : T$. \square

Lemma 2 (Type Substitution Preserves Subtyping). *If $\Delta_1, \bar{X} < \bar{N}, \Delta_2 \vdash S < T$ and $\Delta_1 \vdash \bar{U} < [\bar{U}/\bar{X}]\bar{N}$ with $\Delta_1 \vdash \bar{U}$ ok and none of \bar{X} appearing in Δ_1 , then $\Delta_1, [\bar{U}/\bar{X}]\Delta_2 \vdash [\bar{U}/\bar{X}]S < [\bar{U}/\bar{X}]T$.*

Proof. By induction on the derivation of $\Delta_1, \bar{X} < \bar{N}, \Delta_2 \vdash S < T$. Refer to [1]. \square

Lemma 3 (Type Substitution Preserves Well-formed Instantiation). *If $\Delta_1, \bar{X} < \bar{N}, \Delta_2 \vdash T$ ok and $\Delta_1 \vdash \bar{U} < [\bar{U}/\bar{X}]\bar{N}$ with $\Delta_1 \vdash \bar{U}$ ok and none of \bar{X} appearing in Δ_1 , then $\Delta_1, [\bar{U}/\bar{X}]\Delta_2 \vdash [\bar{U}/\bar{X}]T$ ok.*

Proof. By induction on the derivation of $\Delta_1, \bar{X} < \bar{N}, \Delta_2 \vdash T$ ok, with a case analysis on the last rule used. Refer to [1]. \square

Lemma 4. *Suppose $\Delta_1, \bar{X} < \bar{N}, \Delta_2 \vdash T$ ok and $\Delta_1 \vdash \bar{U} < [\bar{U}/\bar{X}]\bar{N}$ with $\Delta_1 \vdash \bar{U}$ ok and none of \bar{X} appearing in Δ_1 . Then, $\Delta_1, [\bar{U}/\bar{X}]\Delta_2 \vdash \text{bound}_{\Delta_1, [\bar{U}/\bar{X}]\Delta_2}([\bar{U}/\bar{X}]T) < [\bar{U}/\bar{X}](\text{bound}_{\Delta_1, \bar{X} < \bar{N}, \Delta_2}(T))$.*

Proof. Refer to [1]. \square

Lemma 5. *If $\Delta \vdash S < T$ and $\text{fields}(\text{bound}_{\Delta}(T)) = \bar{T} \bar{f}$, then $\text{fields}(\text{bound}_{\Delta}(S)) = \bar{S} \bar{g}$ and $S_i = T_i$ and $g_i = f_i$ for all $i \leq |\bar{f}|$.*

Proof. By straightforward induction on the derivation of $\Delta \vdash S \prec: T$. Refer to [1]. \square

Lemma 6. *If $\Delta \vdash T$ ok and $mtype(m, bound_{\Delta}(T)) = \langle \bar{Y} \langle \bar{P} \rangle \bar{U} \rightarrow U_0$, then for any S such that $\Delta \vdash S \prec: T$ and $\Delta \vdash S$ ok, we have $mtype(m, bound_{\Delta}(S)) = \langle \bar{Y} \langle \bar{P} \rangle \bar{U} \rightarrow U_0'$ and $\Delta, \bar{Y} \langle \bar{P} \rangle \vdash U_0' \prec: U_0$.*

Proof. By straightforward induction on the derivation of $\Delta \vdash S \prec: T$ with a case analysis by the last rule used. Refer to [1]. \square

Lemma 7 (Type Substitution Preserves Typing). *If $\Delta_1, \bar{X} \langle \bar{N} \rangle, \Delta_2; \Gamma \vdash e : T$ and $\Delta_1 \vdash \bar{U} \prec: [\bar{U}/\bar{X}]\bar{N}$ where $\Delta_1 \vdash \bar{U}$ ok and none of \bar{X} appears in Δ_1 , then $\Delta_1, [\bar{U}/\bar{X}]\Delta_2; [\bar{U}/\bar{X}]\Gamma \vdash [\bar{U}/\bar{X}]e : S$ for some S such that $\Delta_1, [\bar{U}/\bar{X}]\Delta_2 \vdash S \prec: [\bar{U}/\bar{X}]T$.*

Proof. By induction on the derivation of $\Delta_1, \bar{X} \langle \bar{N} \rangle, \Delta_2; \Gamma \vdash e : T$ with a case analysis on the last rule used. Refer to [1]. \square

Lemma 8 (Term Substitution Preserves Typing). *If $\Delta; \Gamma, \bar{x} : \bar{T} \vdash e : T$ and $\Delta; \Gamma \vdash \bar{d} : \bar{S}$ where $\Delta \vdash \bar{S} \prec: \bar{T}$, then $\Delta; \Gamma \vdash [\bar{d}/\bar{x}]e : S$ for some S such that $\Delta \vdash S \prec: T$.*

Lemma 9. *If $mtype(m, C \langle \bar{T} \rangle) = \langle \bar{Y} \langle \bar{P} \rangle \bar{U} \rightarrow U$ and $mbody(m \langle \bar{V} \rangle, C \langle \bar{T} \rangle) = \bar{x}. e_0$ where $\Delta \vdash \bar{V}$ ok and $\Delta \vdash \bar{V} \prec: [\bar{V}/\bar{Y}]\bar{P}$ and $\Delta \vdash C \langle \bar{T} \rangle$ ok-inst, then there exist some N and S such that $\Delta \vdash C \langle \bar{T} \rangle \prec: N$ and $\Delta \vdash N$ ok-inst and $\Delta \vdash S \prec: [\bar{V}/\bar{Y}]U$ and $\Delta \vdash S$ ok-inst and $\Delta; \bar{x} : [\bar{V}/\bar{Y}]\bar{U}, \text{this} : selftype(N) \vdash e_0 : S$.*

Proof. By induction of the derivation of $mbody(m \langle \bar{V} \rangle, C \langle \bar{T} \rangle)$ using Lemma 8 with a case analysis on the last rule used. \square

Lemma 10. *Assume class $i_{opt} C \langle \bar{X} \rangle \langle \bar{N} \rangle \langle N \{ \dots \} \rangle$. If $C \langle \bar{X} \rangle \langle \bar{N} \rangle \vdash N$ ok-superclass, then $\bar{X} \langle \bar{N} \rangle \vdash selftype(C \langle \bar{X} \rangle) \prec: selftype(N)$.*

Proof. By the case analysis on the derivation of $C \langle \bar{X} \rangle \langle \bar{N} \rangle \vdash N$ ok-superclass. \square

Lemma 11. *If $\Delta \vdash N \prec: P$, then $\Delta \vdash selftype(N) \prec: selftype(P)$*

Proof. By induction on the derivation of $\Delta \vdash N \prec: P$ using Lemma 10. \square

Proof. (Theorem 4.1) By induction on the derivation of $e \rightarrow e'$ with a case analysis on the reduction rule used. We show only the case of GR-INVK. Other cases can be proved as described in [1].

Case GR-INVK: $e = \text{new } N(\bar{e}) . \langle \bar{V} \rangle m(\bar{d})$ $mbody(m \langle \bar{V} \rangle, N) = \bar{x}. e_0$
 $e' = [\bar{d}/\bar{x}, \text{new } N(\bar{e}) / \text{this}]e_0$

By the rules GT-INVK and GT-NEW, we have

$$\begin{array}{ll} \Delta; \Gamma \vdash \text{new } N(\bar{e}) : N & mtype(m, bound_{\Delta}(N)) = \langle \bar{Y} \langle \bar{P} \rangle \bar{U} \rightarrow U \\ \Delta \vdash \bar{V} \text{ ok} & \Delta \vdash \bar{V} \prec: [\bar{V}/\bar{Y}]\bar{P} \quad \Delta; \Gamma \vdash \bar{d} : \bar{S} \\ \Delta \vdash \bar{S} \prec: [\bar{V}/\bar{Y}]\bar{U} \quad T = [\bar{V}/\bar{Y}]U & \Delta \vdash N \text{ ok} \end{array}$$

By Lemma 9, $\Delta; \bar{x} : [\bar{v}/\bar{y}]\bar{u}, \text{this} : \text{selftype}(\text{P}) \vdash e_0 : \text{S}$ for some P and S such that $\Delta \vdash \text{P}$ ok-inst and $\Delta \vdash \text{N} <: \text{P}$ and $\Delta \vdash \text{S} <: [\bar{v}/\bar{y}]\bar{u}$ and $\Delta \vdash \text{S}$ ok-inst. Since $\text{N} = \text{selftype}(\text{N})$, by Lemma 11 $\Delta \vdash \text{N} <: \text{selftype}(\text{P})$. Then, by the weakening property on type environment and Lemma 8, $\Delta; \Gamma \vdash [\bar{d}/\bar{x}, \text{new } \text{N}(\bar{e})/\text{this}]e_0 : \text{T}_0$ for some T_0 such that $\Delta \vdash \text{T}_0 <: \text{S}$. By S-TRANS, we have $\Delta \vdash \text{T}_0 <: \text{T}$. Finally, letting $\text{T}' = \text{T}_0$ finishes the case. \square

2 Proof of Theorems 4.4 and 4.5

First, we develop a number of the required lemmas.

Lemma 12. *If $\text{fields}(\text{C}) = \bar{\text{T}} \bar{\text{f}}$, then $\text{fields}_{\text{FGJ}}(|\text{C}|) = |\bar{\text{T}}| \bar{\text{f}}$.*

Proof. By induction on the derivation of $\text{fields}(\text{C})$. \square

Lemma 13. *If $\text{classes}(\text{C}) = \bar{\text{E}}$ and $\Delta; \text{C}.E_j \vdash \text{T}$ ok for some Δ , then $[|\bar{\text{U}}|/|\bar{\text{E}}|]|\text{T}| = |\text{T}@U_i|$ for $\bar{\text{U}}$ where $\bar{\text{U}} = \text{F}.\bar{\text{E}}$ or $\bar{\text{U}} = \bar{\text{E}}$.*

Proof. By case analysis on T. \square

Lemma 14. *If $\text{classes}(\text{C}) = \bar{\text{E}}$ and $\emptyset; \text{C} \vdash \text{C}.E_i$ ok and $\text{fields}(\text{C}.E_i) = \bar{\text{T}} \bar{\text{f}}$, then $\text{fields}_{\text{FGJ}}(|\text{C}.E_i| < |\bar{\text{U}}| >) = |\bar{\text{T}}@U_i| \bar{\text{f}}$ for $\bar{\text{U}}$ where $\bar{\text{U}} = \text{F}.\bar{\text{E}}$ or $\bar{\text{U}} = \bar{\text{E}}$.*

Proof. By induction on the derivation of $\emptyset; \text{C} \vdash \text{C}.E_i$ ok. \square

Lemma 15. *If $\text{mtype}(\text{m}, \text{C}) = <\bar{\text{X}}<\bar{\text{C}}>\bar{\text{T}} \rightarrow \text{T}_0$, then $\text{mtype}_{\text{FGJ}}(\text{m}, \text{C}) = <|\bar{\text{X}}<\bar{\text{C}}|>|\bar{\text{T}}| \rightarrow |\text{T}_0|$.*

Proof. By induction on the derivation of $\text{mtype}(\text{m}, \text{C})$. \square

Lemma 16. *If $\text{classes}(\text{C}) = \bar{\text{E}}$ and $\emptyset; \text{C} \vdash \text{C}.E_i$ ok and $\text{mtype}(\text{m}, \text{C}.E_i) = <\bar{\text{X}}<\bar{\text{C}}>\bar{\text{T}} \rightarrow \text{T}_0$, then $\text{mtype}_{\text{FGJ}}(\text{m}, |\text{C}.E_i| < |\bar{\text{U}}| >) = <|\bar{\text{X}}<\bar{\text{C}}|>|\bar{\text{T}}@U_i| \rightarrow |\text{T}_0@U_i|$ for $\bar{\text{U}}$ where $\bar{\text{U}} = \text{F}.\bar{\text{E}}$ or $\bar{\text{U}} = \bar{\text{E}}$.*

Proof. By induction on the derivation of $\emptyset; \text{C} \vdash \text{C}.E_i$ ok. \square

Lemma 17. *If $\Delta; \text{C} \vdash \text{F}, \text{C}$ ok and $\Delta \vdash \text{F} <: \text{C}$, then $|\Delta| \vdash_{\text{FGJ}} |\text{F}|_{\text{C}}$ ok.*

Proof. Easy. \square

Lemma 18. *If $\Delta \vdash \text{S} <: \text{T}$, then $|\Delta| \vdash_{\text{FGJ}} |\text{S}| <: |\text{T}|$.*

Proof. By induction on the derivation of $\Delta \vdash \text{S} <: \text{T}$. \square

Lemma 19. *If $\text{X} <: \text{C}; \text{A} \vdash \text{T}$ ok, then $[|\text{F}/\text{X}|]\text{T} = [|\text{F}|_{\text{C}}/|\text{X}|_{\text{C}}]|\text{T}|$.*

Proof. By case analysis on T. \square

Lemma 20. *If $\Delta \vdash \text{F} <: \text{C}$ and $\text{classes}(\text{C}) = \bar{\text{E}}$, then $|\Delta| \vdash_{\text{FGJ}} |\text{F}|_{\text{C}} <: ([|\text{F}|_{\text{C}}/|\text{X}|_{\text{C}}]|\text{C}.E| < |\text{X}.\bar{\text{E}}| >), \text{C}$.*

Proof. By case analysis on F. \square

Lemma 21. *If $\Delta; \mathbf{C} \vdash \mathbf{T}$ ok and $|\Delta| \vdash_{\text{FGJ}} \mathbf{T}$ ok. Similarly, if $\text{classes}(\mathbf{C}) = \bar{\mathbf{E}}$ and $\Delta; \mathbf{C}.\mathbf{E}_i \vdash \mathbf{T}$ ok, then $|\Delta|, |\langle \mathbf{C} \rangle| \vdash_{\text{FGJ}} |\mathbf{T}|$ ok.*

Proof. By case analysis on \mathbf{T} . □

Lemma 22. *If $\text{classes}(\mathbf{C}) = \bar{\mathbf{E}}$, then $|\mathbf{X} \langle \mathbf{C} \rangle; |\mathbf{X}.\mathbf{E}_i| \vdash_{\text{FGJ}} [\mathbf{C}.\mathbf{E}_i] \langle |\mathbf{X}.\bar{\mathbf{E}}| \rangle$ ok-bound.*

Proof. Easily proved by WFB-CLASSSELF. □

Lemma 23. *If $\text{classes}(\mathbf{C}) = \bar{\mathbf{E}}$ and $\emptyset; \mathbf{A} \vdash \mathbf{C}.\mathbf{E}_i$ ok, then $\text{fix}(\mathbf{C}, \mathbf{E}_i) = \text{class } |\mathbf{C}.\mathbf{E}_i| \langle [\mathbf{C}.\mathbf{E}_i] \langle |\mathbf{C}.\bar{\mathbf{E}}| \rangle \{ \} \rangle$ ok.*

Proof. Can be proved by GT-CLASS. □

Proof. (Theorem 4.4) We prove the theorem in two steps: first, it is shown that if $\Delta; \Gamma; \mathbf{C} \vdash \mathbf{e}:\mathbf{T}$ then $|\Delta|; |\Gamma| \vdash_{\text{FGJ}} |\mathbf{e}|_{\Delta, \Gamma, \mathbf{C}}:|\mathbf{T}|$, and if $\Delta; \Gamma; \mathbf{C}.\mathbf{E} \vdash \mathbf{e}:\mathbf{T}$ then $|\Delta|, |\langle \mathbf{C} \rangle; |\Gamma| \vdash_{\text{FGJ}} |\mathbf{e}|_{\Delta, \Gamma, \mathbf{C}.\mathbf{E}}:|\mathbf{T}|$; and second, we show CT is ok.

The first part is proved by induction on the derivation of $\Delta; \Gamma; \mathbf{C} \vdash \mathbf{e}:\mathbf{T}$ and $\Delta; \Gamma; \mathbf{C}.\mathbf{E} \vdash \mathbf{e}:\mathbf{T}$. We show only the latter case since the former can be proved similarly to the latter.

Case T-FIELD: $\mathbf{e} = \mathbf{e}_0.\mathbf{f}_i$ $\Delta; \Gamma; \mathbf{C}.\mathbf{E} \vdash \mathbf{e}_0:\mathbf{T}_0$
 $\text{fields}(\text{bound}_{\Delta}(\mathbf{T}_0 @ \mathbf{C}.\mathbf{E})) = \bar{\mathbf{T}} \bar{\mathbf{f}}$ $\mathbf{T} = \mathbf{T}_i @ \mathbf{T}_0$

By induction hypothesis, we have $|\Delta|, |\langle \mathbf{C} \rangle; |\Gamma| \vdash_{\text{FGJ}} |\mathbf{e}_0|_{\Delta, \Gamma, \mathbf{C}.\mathbf{E}}:|\mathbf{T}_0|$. Case analysis on \mathbf{T}_0 .

Subcase: $\mathbf{T}_0 = \mathbf{P}$

By Lemma 12 and the fact that $|\Delta(\mathbf{P})| = |\Delta|(|\mathbf{P}|)$, $\text{fields}_{\text{FGJ}}(\text{bound}_{(|\Delta|, |\langle \mathbf{C} \rangle)}(|\mathbf{P}|)) = |\bar{\mathbf{T}}| \bar{\mathbf{f}}$. Since $\mathbf{T}_i @ \mathbf{T}_0 = \mathbf{T}_i$, by GT-FIELD $|\Delta|, |\langle \mathbf{C} \rangle; |\Gamma| \vdash_{\text{FGJ}} |\mathbf{e}_0|_{\Delta, \Gamma, \mathbf{C}.\mathbf{E}}.\mathbf{f}_i:|\mathbf{T}_i|$.

Subcase: $\mathbf{T}_0 = \mathbf{C}' . \mathbf{D}_j$ where $\text{classes}(\mathbf{C}') = \bar{\mathbf{D}}$

Since there is $\text{class } \mathbf{C}' \mathbf{D}_i \text{Fix } \langle [\mathbf{C}' . \mathbf{D}_j] \langle |\mathbf{C}' . \bar{\mathbf{D}}| \rangle \{ \} \rangle$, by Lemma 13 $\text{fields}_{\text{FGJ}}(\mathbf{C}' . \mathbf{D}_j) = \text{fields}_{\text{FGJ}}(\mathbf{C}' \mathbf{D}_i \text{Fix}) = |\bar{\mathbf{T}} @ \mathbf{C}' . \mathbf{D}_j| \bar{\mathbf{f}}$. By GT-FIELD, $|\Delta|, |\langle \mathbf{C} \rangle; |\Gamma| \vdash_{\text{FGJ}} |\mathbf{e}_0|_{\Delta, \Gamma, \mathbf{C}.\mathbf{E}}.\mathbf{f}_i:|\mathbf{T}_i @ \mathbf{C}' . \mathbf{D}_j|$.

Subcase: $\mathbf{T}_0 = \mathbf{X} . \mathbf{D}_j$ where $\mathbf{X} \langle \mathbf{C}' \rangle \in \Delta$ and $\text{classes}(\mathbf{C}') = \bar{\mathbf{D}}$

Since $\text{fields}(\text{bound}_{\Delta}(\mathbf{X} . \mathbf{D}_j)) = \text{fields}(\mathbf{C}' . \mathbf{D}_j)$ by .FJ typing rule, by Lemma 13 $\text{fields}_{\text{FGJ}}(\text{bound}_{(|\Delta|, |\langle \mathbf{C} \rangle)}(|\mathbf{X} . \mathbf{D}_j|)) = \text{fields}_{\text{FGJ}}([\mathbf{C}' . \mathbf{D}_j] \langle |\mathbf{X} . \bar{\mathbf{D}}| \rangle) = |\bar{\mathbf{T}} @ \mathbf{X} . \mathbf{D}_j|$. By GT-FIELD, $|\Delta|, |\langle \mathbf{C} \rangle; |\Gamma| \vdash_{\text{FGJ}} |\mathbf{e}_0|_{\Delta, \Gamma, \mathbf{C}.\mathbf{E}}.\mathbf{f}_i:|\mathbf{T}_i @ \mathbf{X} . \mathbf{D}_j|$.

Subcase: $\mathbf{T}_0 = .\mathbf{E}_j$ where $\text{classes}(\mathbf{C}) = \bar{\mathbf{E}}$

Since $\text{fields}(. \mathbf{E}_j @ \mathbf{C}.\mathbf{E}) = \text{fields}(\mathbf{C} . \mathbf{E}_j)$, by Lemma 13 $\text{fields}_{\text{FGJ}}(\text{bound}_{(|\Delta|, |\langle \mathbf{C} \rangle)}(\mathbf{E}_j)) = \text{fields}_{\text{FGJ}}([\mathbf{C} . \mathbf{E}_j] \langle |\bar{\mathbf{E}}| \rangle) = |\bar{\mathbf{T}} @ . \mathbf{E}_j| \bar{\mathbf{f}}$. By GT-FIELD, $|\Delta|, |\langle \mathbf{C} \rangle; |\Gamma| \vdash_{\text{FGJ}} |\mathbf{e}_0|_{\Delta, \Gamma, \mathbf{C}.\mathbf{E}}.\mathbf{f}_i:|\mathbf{T}_i @ . \mathbf{E}_j|$.

Case GT-INVK, GT-NEW:

Similar to the case above. The second part ($|CT|$ ok) follows from the first part with examination of the rules GT-METHOD, GT-CLASS and GT-CLASSSELF. It is easy to show that:

1. if $\mathbf{C} \vdash \mathbf{M}$ ok, then $\mathbf{C} \vdash_{\text{FGJ}} |\mathbf{M}|_{\mathbf{C}}$ ok.

2. if $\mathbf{C}.E_i \vdash \mathbf{M}$ ok where $\text{classes}(\mathbf{C}) = \bar{\mathbf{E}}$, then $\mathbf{C}\$E_i \langle \langle \mathbf{C} \rangle \rangle \vdash_{\text{FGJ}} |\mathbf{M}|_{\mathbf{C}.E_i}$.
3. if $\mathbf{C} \vdash \mathbf{NL}$ ok, then $\vdash_{\text{FGJ}} |\mathbf{NL}|_{\mathbf{C}}$ ok.
4. if $\vdash \mathbf{C}$ ok, then $\vdash_{\text{FGJ}} |\mathbf{C}|$ ok.

□

The following three lemmas from [2] are ones used to prove .FJ subject reduction. We write $\mathbf{A} \langle \# \mathbf{B} \rangle$ if either (1) $\mathbf{A} = \mathbf{C}$, $\mathbf{B} = \mathbf{D}$, and $\vdash \mathbf{C} \prec \mathbf{D}$, or (2) $\mathbf{A} = \mathbf{C}.E$, $\mathbf{B} = \mathbf{D}.E$, and $\vdash \mathbf{C} \prec \mathbf{D}$.

Lemma 24. *If $\mathbf{A} \langle \# \mathbf{B} \rangle$ and $\text{mtype}(\mathbf{m}, \mathbf{B}) = \langle \bar{\mathbf{X}} \langle \bar{\mathbf{C}} \rangle \bar{\mathbf{U}} \rightarrow \mathbf{U}_0 \rangle$, then $\text{mtype}(\mathbf{m}, \mathbf{A}) = \langle \bar{\mathbf{X}} \langle \bar{\mathbf{C}} \rangle \bar{\mathbf{U}} \rightarrow \mathbf{U}_0 \rangle$.*

Lemma 25. *If $\Delta, \bar{\mathbf{X}} \langle \bar{\mathbf{C}} \rangle; \Gamma; \mathbf{A} \vdash \mathbf{e} : \mathbf{T}$ and $\Delta \vdash \bar{\mathbf{F}}$ ok and $\Delta \vdash \bar{\mathbf{F}} \prec \bar{\mathbf{C}}$, then there exists some \mathbf{S} such that $\Delta; [\bar{\mathbf{F}}/\bar{\mathbf{X}}]\Gamma; \mathbf{A} \vdash [\bar{\mathbf{F}}/\bar{\mathbf{X}}]\mathbf{e} : \mathbf{S}$ and $\Delta \vdash \mathbf{S} \prec [\bar{\mathbf{F}}/\bar{\mathbf{X}}]\mathbf{T}$.*

Lemma 26. *If $\Delta; \Gamma, \bar{\mathbf{x}} : \bar{\mathbf{T}}; \mathbf{A} \vdash \mathbf{e} : \mathbf{T}$ and $\Delta; \Gamma; \mathbf{A} \vdash \bar{\mathbf{d}} : \bar{\mathbf{S}}$ and $\Delta \vdash \bar{\mathbf{S}} \prec \bar{\mathbf{T}}$, then there exists some \mathbf{S} such that $\Delta; \Gamma; \mathbf{A} \vdash [\bar{\mathbf{d}}/\bar{\mathbf{x}}]\mathbf{e} : \mathbf{S}$ and $\Delta \vdash \mathbf{S} \prec \mathbf{T}$.*

Lemma 27. *If $\bar{\mathbf{X}} \langle \mathbf{C} \rangle \in \Delta$ and $\Delta; \Gamma; \mathbf{A} \vdash \mathbf{e} : \mathbf{T}$ and $\Delta \vdash \mathbf{F} \prec \mathbf{C}$, then $||[\bar{\mathbf{F}}/\bar{\mathbf{X}}]\mathbf{e}|_{\Delta, \Gamma, \mathbf{A}} = ||[\mathbf{F}|_{\mathbf{C}}/|\bar{\mathbf{X}}|_{\mathbf{C}}]|\mathbf{e}|_{\Delta, \Gamma, \mathbf{A}}$*

Proof. By induction on the derivation of $\Delta; \Gamma; \mathbf{A} \vdash \mathbf{e} : \mathbf{T}$. Note that if $\mathbf{C} \prec \mathbf{D}$, then $|\mathbf{F}|_{\mathbf{D}} = ||[\mathbf{F}|_{\mathbf{C}}/|\bar{\mathbf{X}}|_{\mathbf{C}}]|\mathbf{X}|_{\mathbf{D}}$. □

Lemma 28. *If $\Delta; \Gamma; \mathbf{A} \vdash \bar{\mathbf{e}} : \bar{\mathbf{T}}$ and $\Delta'; \Gamma'; \mathbf{A}_0 \vdash \mathbf{e}_0 : \mathbf{T}_0$, then $||[\bar{\mathbf{e}}/\bar{\mathbf{x}}]\mathbf{e}_0|_{\Delta, \Gamma, \mathbf{A}} = ||[\bar{\mathbf{e}}|_{\Delta, \Gamma, \mathbf{A}}/\bar{\mathbf{x}}]|\mathbf{e}_0|_{\Delta', \Gamma', \mathbf{A}_0}$*

Proof. By induction on the derivation of $\Delta'; \Gamma'; \mathbf{A}_0 \vdash \mathbf{e}_0 : \mathbf{T}_0$. □

Proof. (Theorem 4.5) By induction on the derivation of $\mathbf{e} \rightarrow \mathbf{e}'$ with a case analysis on the last reduction rule used.

Case R-FIELD: $\mathbf{e} = \text{new } \mathbf{A}_0(\bar{\mathbf{e}}) . \mathbf{f}_i \quad \mathbf{e}' = \mathbf{e}_i \quad \text{fields}(\mathbf{A}_0) = \bar{\mathbf{T}} \bar{\mathbf{f}}$

Since $|\text{new } \mathbf{A}_0(\bar{\mathbf{e}}) . \mathbf{f}_i|_{\Delta, \Gamma, \mathbf{A}} = \text{new } |\mathbf{A}_0|(|\bar{\mathbf{e}}|_{\Delta, \Gamma, \mathbf{A}}) . \mathbf{f}_i$, $\text{fields}_{\text{FGJ}}(|\mathbf{A}_0|) = |\bar{\mathbf{T}}@_{\mathbf{A}_0}| \bar{\mathbf{f}}$ by Lemmas 12 and 13. By GR-FIELD, $\text{new } |\mathbf{A}_0|(|\bar{\mathbf{e}}|_{\Delta, \Gamma, \mathbf{A}}) . \mathbf{f}_i \rightarrow |\mathbf{e}_i|$

Case R-INVK: $\mathbf{e} = \text{new } \mathbf{A}_0(\bar{\mathbf{e}}) . \langle \bar{\mathbf{P}} \rangle_{\mathbf{m}}(\bar{\mathbf{d}}) \quad \mathbf{e}' = [\bar{\mathbf{d}}/\bar{\mathbf{x}}, \text{new } \mathbf{A}_0(\bar{\mathbf{e}})/\text{this}]\mathbf{e}'_0$
 $\text{mbody}(\mathbf{m} \langle \bar{\mathbf{P}} \rangle, \mathbf{A}_0) = \bar{\mathbf{x}} . \mathbf{e}'_0$

Assume that $\langle \bar{\mathbf{X}} \langle \bar{\mathbf{C}} \rangle \mathbf{T}_0 \mathbf{m}(\bar{\mathbf{T}} \bar{\mathbf{x}}) \{ \text{return } \mathbf{e}_0; \}$. By the definition of *mbody*, $\mathbf{e}'_0 = [\bar{\mathbf{P}}/\bar{\mathbf{X}}]\mathbf{e}_0$. By the definition of the translation, $|\text{new } \mathbf{A}_0(\bar{\mathbf{e}}) . \langle \bar{\mathbf{P}} \rangle_{\mathbf{m}}(\bar{\mathbf{d}})|_{\Delta, \Gamma, \mathbf{A}} = \text{new } |\mathbf{A}_0|(|\bar{\mathbf{e}}|_{\Delta, \Gamma, \mathbf{A}}) . \langle |\bar{\mathbf{P}}|_{\bar{\mathbf{C}}} \rangle_{\mathbf{m}}(|\bar{\mathbf{d}}|_{\Delta, \Gamma, \mathbf{A}})$. Let $\Delta' = \bar{\mathbf{X}} \langle \bar{\mathbf{C}} \rangle$, $\Gamma' = \bar{\mathbf{x}} : \bar{\mathbf{T}}, \text{this} : \mathbf{A}_0$. Since $\text{mbody}_{\text{FGJ}}(\mathbf{m} \langle |\bar{\mathbf{P}}|_{\bar{\mathbf{C}}} \rangle, |\mathbf{A}_0|) = \bar{\mathbf{x}} . \mathbf{e}''_0$ where $\mathbf{e}''_0 = ||[\bar{\mathbf{P}}|_{\bar{\mathbf{C}}}/|\bar{\mathbf{X}}|_{\bar{\mathbf{C}}}]|\mathbf{e}_0|_{\Delta', \Gamma', \mathbf{A}_0}$, we must show that $||[\bar{\mathbf{d}}/\bar{\mathbf{x}}, \text{new } \mathbf{A}_0(\bar{\mathbf{e}})/\text{this}]\mathbf{e}'_0|_{\Delta, \Gamma, \mathbf{A}} = ||[\bar{\mathbf{d}}|_{\Delta, \Gamma, \mathbf{A}}/\bar{\mathbf{x}}, \text{new } |\mathbf{A}_0|(|\bar{\mathbf{e}}|_{\Delta, \Gamma, \mathbf{A}})/\text{this}]\mathbf{e}''_0$.

$$\begin{aligned}
& ||[\bar{\mathbf{d}}|_{\Delta, \Gamma, \mathbf{A}}/\bar{\mathbf{x}}, \text{new } |\mathbf{A}_0|(|\bar{\mathbf{e}}|_{\Delta, \Gamma, \mathbf{A}})/\text{this}]\mathbf{e}''_0 && \text{(by Lemma 27)} \\
& = ||[\bar{\mathbf{d}}|_{\Delta, \Gamma, \mathbf{A}}/\bar{\mathbf{x}}, \text{new } |\mathbf{A}_0|(|\bar{\mathbf{e}}|_{\Delta, \Gamma, \mathbf{A}})/\text{this}][[\bar{\mathbf{P}}/\bar{\mathbf{X}}]\mathbf{e}_0|_{\Delta', \Gamma', \mathbf{A}_0}] && \text{(by Lemma 28)} \\
& = ||[\bar{\mathbf{d}}/\bar{\mathbf{x}}, \text{new } \mathbf{A}_0(\bar{\mathbf{e}})/\text{this}][[\bar{\mathbf{P}}/\bar{\mathbf{X}}]\mathbf{e}_0|_{\Delta, \Gamma, \mathbf{A}}].
\end{aligned}$$

Case RC-FIELD, RC-INVK-RECV, RC-INVK-ARG, RC-NEW-ARG:

Immediate from the induction hypothesis. □

References

1. Atsushi Igarashi, Benjamin C. Pierce, and Philip Wadler. Featherweight Java: A minimal core calculus for Java and GJ. *ACM Transactions on Programming Languages and Systems*, 23(3):396–450, May 2001. A preliminary summary appeared in *Proceedings of the ACM Conference on Object-Oriented Programming, Systems, Languages, and Applications (OOPSLA '99)*, ACM SIGPLAN Notices, volume 34, number 10, pages 132–146, Denver, CO, October 1999.
2. Chieri Saito, Atsushi Igarashi, and Mirko Viroli. Lightweight family polymorphism. *Journal of Functional Programming*, 2008. To appear. A preliminary summary appeared in *Proceedings of the 3rd Asian Symposium on Programming Languages and Systems (APLAS2005)*, Springer LNCS vol. 3780, pages 161–177, November, 2005.